Projected Inverse Optimal Control : a unifying vision of Inverse Optimal Control solution methods

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JNRH 2023

Inverse optimal control for human behavior understanding

- Context: human-robot interaction to better understand human behavior.
- Hypothesis: human create its movement by optimizing weighted functions. [1]

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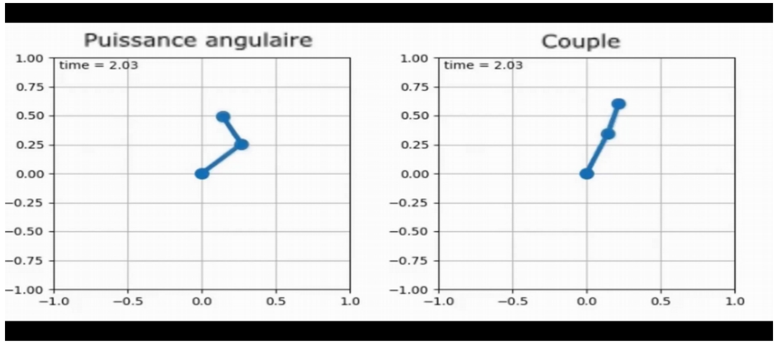

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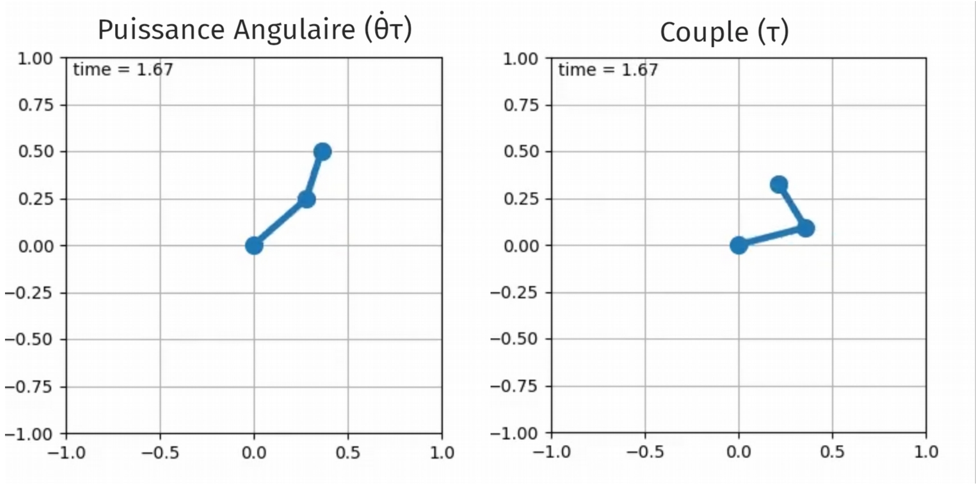

- <u>Problem</u>: Uncertainties on human motion measurements for IOC resolutions.

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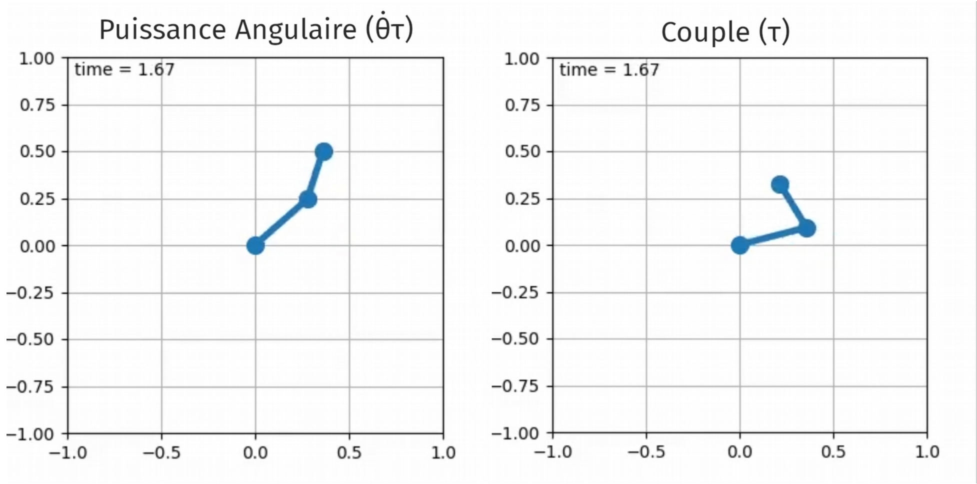

- Problem: Uncertainties on human motion measurements for IOC resolutions.

- Contributions:
  - Introduce an approach, **Projected Inverse Optimal Control (PIOC)** [2]**:**
    - holistic approach to better understand resolutions proposed in litterature;
    - made for trajectories with uncertainties;
    - Lead to discussions (identifiabiliy, dimensionality)

1. Berret et al., (2011). “Evidence for Composite Cost Functions in Arm Movement Planning: An Inverse Optimal Control Approach,” PLOS Computational Biology
2. Colombel et al., (2023). “Holistic view of Inverse Optimal Control by introducing projections on singularity curves”, ICRA

Direct Optimal Control and resolutions of IOC

Direct Optimal Control

DOC generates optimal trajectories s^* from the basis C of cost functions under constraints f and h:

$$\mathrm{s}^* = \arg\min_{\mathrm{s}} C(\mathrm{s})$$

$$\textbf{s.t. } f_{1,\ldots,n_f}(\mathrm{s}) = 0,\ h_{1,\ldots,n_h}(\mathrm{s}) \leq 0$$

The trajectory is $\mathrm{s} = [\mathrm{s}_1, \ldots, \mathrm{s}_{n_t}]^T$ and the basis :

$$C(\mathrm{s}) = \sum_{k=1}^{n_c} \omega_k . C_k$$

With ω_k positive for k=1..n_c (number of cost functions) and C_k cost functions such as: torque, jerk, acceleration, angular power, etc

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Inverse Optimal Control

- Bilevel [1]

$$\min_{\omega} \quad \|s^* - s_M\|^2$$

$$\textbf{with } s^* = \operatorname{argmin}_{s} \sum_{k=1}^{n_c} \omega_k \sum_{t=1}^{n_t} C_k(s_t)$$

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- Approximately Inverse Optimal Control [2]

$$\frac{\partial L}{\partial \mathbf{s}}(\mathbf{s}^*) = \sum_{k=1}^{n_c} \omega_k \frac{\partial C_k}{\partial \mathbf{s}}(\mathbf{s}^*) + \sum_{i=1}^{n_f} \lambda_i \frac{\partial f_i}{\partial \mathbf{s}}(\mathbf{s}^*) + \sum_{j=1}^{n_h} \nu_j \frac{\partial h_j}{\partial \mathbf{s}}(\mathbf{s}^*) = \mathbf{0}$$

$f_i(\mathrm{s}^*) = 0, \quad i =$

$h_j(\mathrm{s}^*) \leq 0, \quad j =$ ... y

$\nu_j \geq 0, \quad j =$

$\nu_j h_j(\mathrm{s}^*) = 0, \quad j =$ ... slackness

$$\underbrace{[\mathbf{J}_\omega, \mathbf{J}_\lambda, \mathbf{J}_\nu]}_{\mathbf{J}} \underbrace{\begin{pmatrix} \omega \\ \lambda \\ \nu \end{pmatrix}}_{\mathbf{z}} = \mathbf{0}$$

1. Mombaur, et al., (2010). "From human to humanoid locomotion—an inverse optimal control approach," Autonomous Robots
2. Lin, et al., (2016) "Human motion segmentation using cost weights recovered from inverse optimal control," Humanoids

Problem statement

Uncertainties

PIOC : parametrization and decoupling conditions

- Polynomial parametrization:
  - Reduce number of parameters
  - Reduce computing time
  - Simplify the state $\theta_1(t) = \sum_{k=0}^{n_d} \alpha_k t^k.$
- Thanks to constraints, it can be expressed with a minimal number of parameters [1]. For example :

$$\theta_1 \to \alpha_6 \text{ et } \theta_2 \to \beta_6$$

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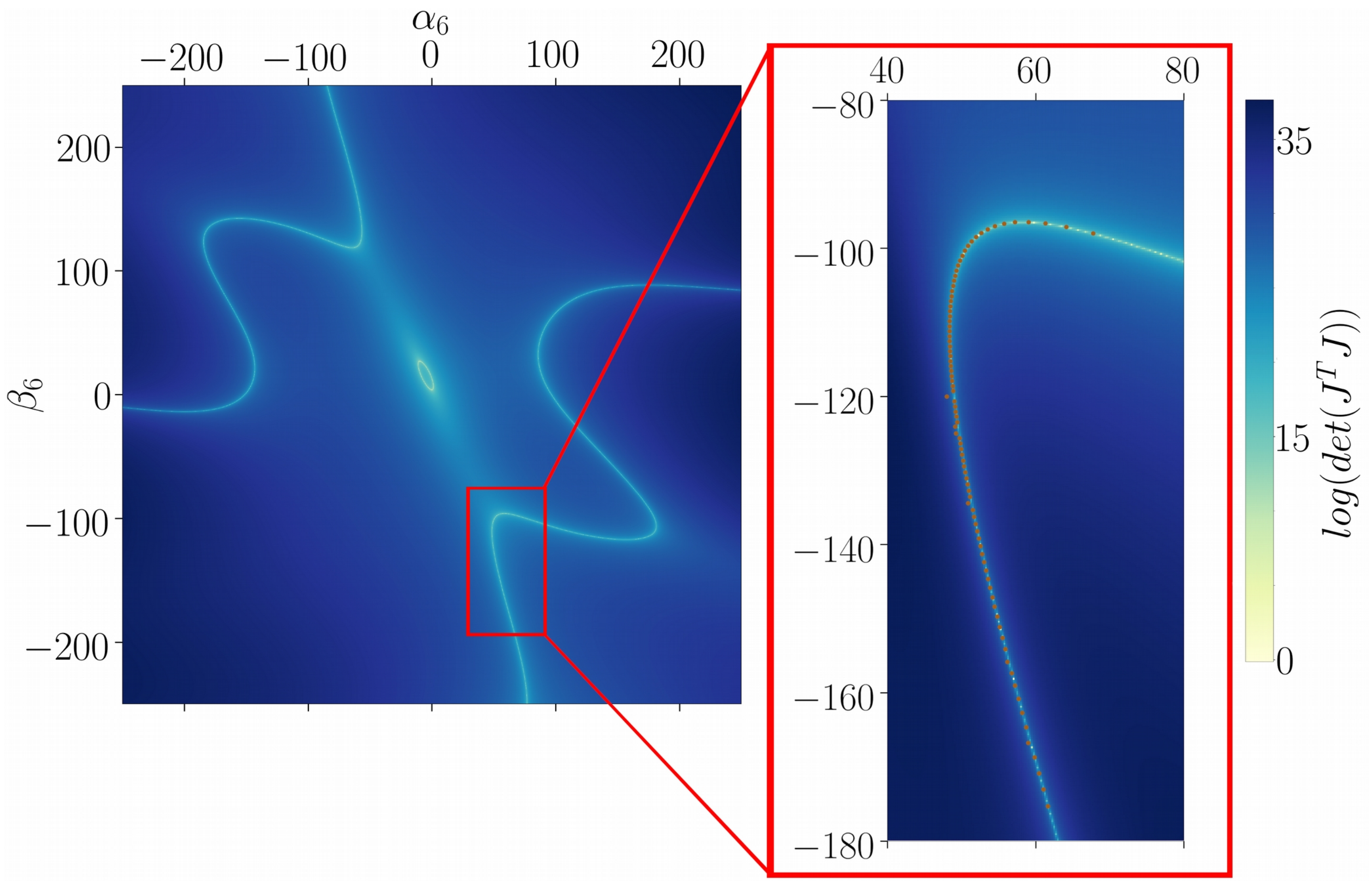
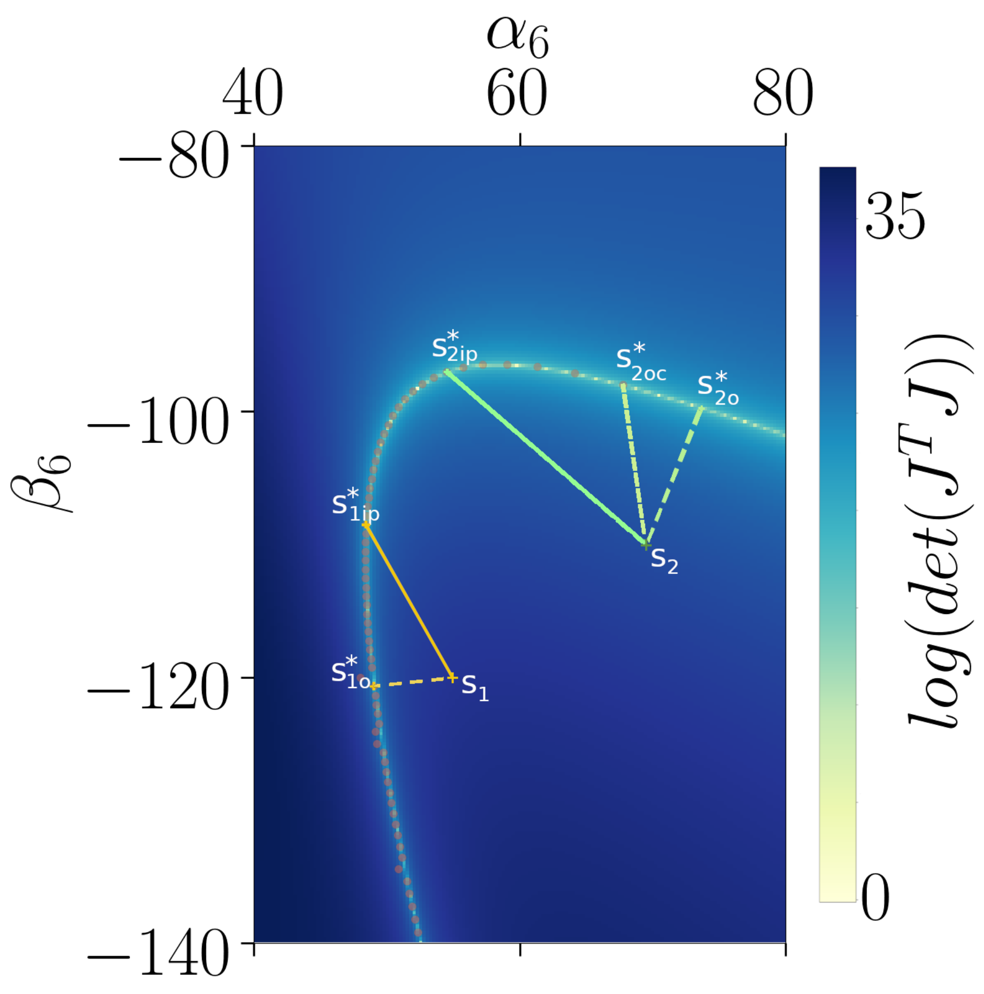
- Conditions can be use to analyse [2] :
  - the problem (identifiability, dimensionality, optimality)
  - the solution (reliability).

1. Colombel et al., (2023). "Holistic view of Inverse Optimal Control by introducing projections on singularity curves", ICRA
2. Colombel et al., (2022). "On the Reliability of Inverse Optimal Control", ICRA

PIOC : Singularity curves

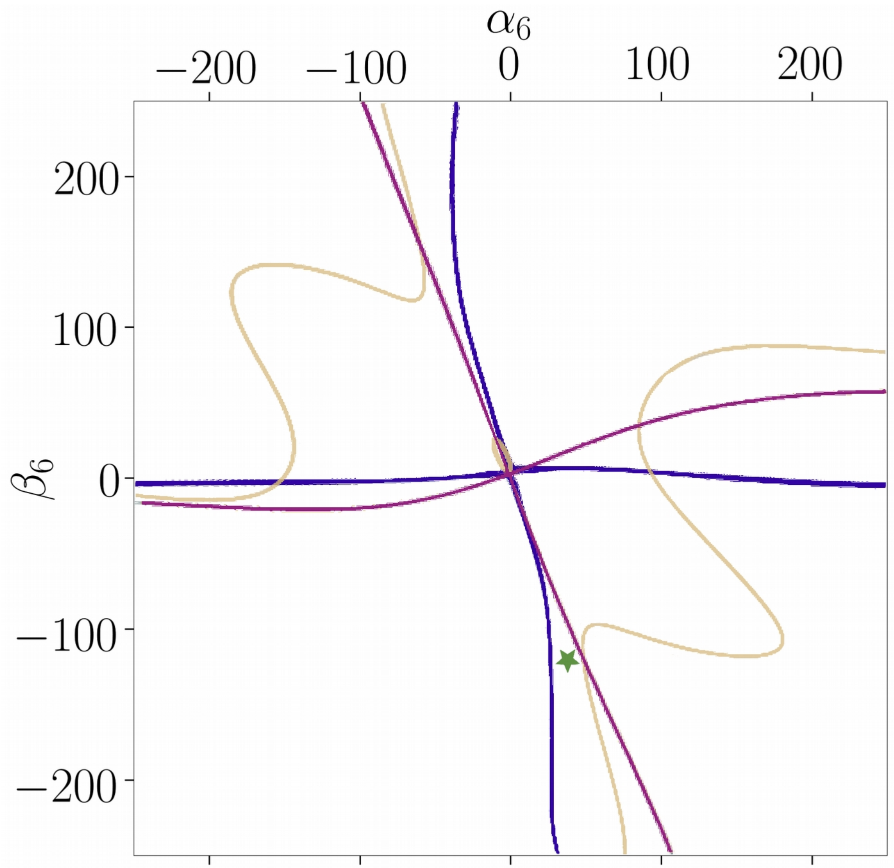


PIOC: Projections



Conclusion

- Introduce an approach, **Projected Inverse Optimal Control**, based on:
  - parametrization of measurement states and its impact on
  - decoupling of conditions: identifiability, singularity and feasibility conditions;
  - illustration of the choice of solution by a projection;
- Openings:
  - dimensionality and identifiability of the problem;
  - solution for the choice of basis.



Thank you.